2 - harder

Kinetic Theory - Clark Maxwell - harder

A Physics

1. State the assumptions of James Clark Maxwell's kinetic theory. (3)

2. Maxwell's final formula is....

$$PV = \tfrac{1}{3} N m \langle c^2 \rangle \quad \text{— formula 1.}$$

(a) what does each symbol mean? (5)

(b) Can you write, without your notes, how you get from this to...

$$P = \tfrac{1}{3} \rho \langle c^2 \rangle$$

(c) Can you explain, without your notes, how you get from the basic assumptions to formula 1. ? (a lot.)

Key formulas...
- $PV = \tfrac{1}{3} N m \langle c^2 \rangle$
- $P = \tfrac{1}{3} \rho \langle c^2 \rangle$
- Internal energy $E = \tfrac{3}{2} RT$ Joules/mole
- Avogadro's number $N_A = 6 \times 10^{23}$.
- mass of proton = mass of neutron $= 1.7 \times 10^{-27}$ kg.
- Volume of a sphere $V = \tfrac{4}{3}\pi r^3$, m^3
- 1 atm. $= 1 \times 10^5$ N/m²
- $R = 8.3$ J/mole/K.

3. A spherical balloon, radius 15 cm, contains helium gas (relative atomic mass 4) at 1.5 atmospheres, and 27°C.

a) Calculate.... the no. of moles of helium (3).

b) .... the no. of helium atoms. (2)

c) ... the r.m.s. speed of the helium atoms (3)

d) ... the internal energy of the helium in the balloon. (3)

The gas is cooled in the balloon. Calculate its new internal energy and r.m.s. speed at...

e) - 73°C (5) and (f) at - 150°C (3)

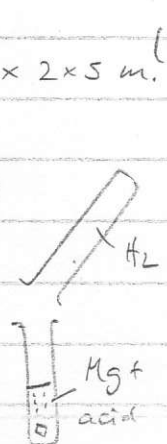
5/m 4. The density of air at 1 atmosphere and 20°C is 1.3 kg/m³

4/10 a) Calculate the total internal energy of the air in a room 3 × 2 × 5 m. (4)

b) Calculate the r.m.s. speed of the air molecules (2)

2 3/10 c) Why are we normally unaware of ... the air's internal energy and ... the air particles' high speed? (4)

5. Hydrogen has a density of 0.09 kg/m³ at 1atm. and 27°C. What is the r.m.s. speed of hydrogen molecules? (3)



TN

3.00 + 10.13

Kinetic Theory – Clerk Maxwell Answers A Physics

1. – atoms in constant random motion, elastic collisions
   – atoms have negligible (almost zero) physical size
   – atoms do not attract each other /3

2. – See your notes!

3. a) n moles use $PV = nRT$ ① $V = \frac{4}{3}\pi r^3 = 0.0141\ m^3$ ①
   so $n = PV/RT = 1.5 \times 1\times10^5 \times 0.0141 / 8.3 \times 300 = 0.852$ mol ①

   b) no. of He atoms $= 0.852 \times 6\times10^{23}$ ① $= 5.11 \times 10^{23}$ atoms ①

   c) r.m.s. speed ... use $PV = \frac{1}{3} N m \langle c^2 \rangle$ ①
   He is 2 protons + 2 neutrons $\rightarrow$ mass $= 4 \times 1.7\times10^{-27} = 6.8\times10^{-27}$ ①
   so $1.5\times10^5 \times 0.0141 = \frac{1}{3} \times 5.11\times10^{23} \times 6.8\times10^{-27} \times \langle c^2 \rangle$
   so $\langle c^2 \rangle = 1.826 \times 10^6$
   r.m.s. speed $= \sqrt{\langle c^2 \rangle} = 1351$ ① m/s

   d) Internal energy $= \frac{3}{2} RT$ ① J/mole ← moles!
   $= \frac{3}{2} \times 8.3 \times 300 \times 0.85$ ① Joules
   $= 3182$ ① Joules

   e) Internal energy $= \frac{3}{2} RT \times n$ ① J so $E \propto T$ ① ← in kelvin.
   $-73°C$ = 200K is $\frac{2}{3}$ of 300K
   so int. energy is $\frac{2}{3}$ of $3182 = 2121$ Joules ①
   Int. energy $= \frac{1}{2} Nm \langle c^2 \rangle$ so $\langle c^2 \rangle$ is $\frac{2}{3}$ of $1.826\times10^6 = 1.217\times10^6$
   so r.m.s. speed $\sqrt{\langle c^2 \rangle} = \sqrt{1.217\times10^6} = 1103$ m/s ①
   at $-150°C$ = 123K ... energy is $\frac{123}{300} \times 3182 = 1304$ J ①
   mean square speed $\langle c^2 \rangle = \frac{123}{300} \times 1.826\times10^6 = 7.49\times10^5$ ①
   so r.m.s. speed $\sqrt{\langle c^2 \rangle} = 865$ m/s ①

4. a) $PV = nRT$ ① $\rightarrow$ n moles $= PV/RT = 1\times10^5 \times (3 \times 2 \times 5) / 8.3 \times 293 = 1234$ moles ①
   Int. energy $E = \frac{3}{2} RT \times n$ ① $= 4.5\times10^6$ J ① $= \frac{3}{2} PV$!!

   b) use $P = \frac{1}{3} \rho \langle c^2 \rangle$ ① $\rightarrow \langle c^2 \rangle = 3P/\rho = 3\times1\times10^5/1.3 = 2.31\times10^5$
   so r.m.s speed $\sqrt{\langle c^2 \rangle} = 430$ m/s ①

   c) molecules are tiny ... v. frequent impacts ... constant rate ... random motion

5. $P = \frac{1}{3} \rho \langle c^2 \rangle$ ① $\rightarrow \langle c^2 \rangle = 3P/\rho = 3\times10^5/0.09 = 3.33\times10^6$ ①
   r.m.s speed $= \sqrt{\langle c^2 \rangle} = 1825$ m/s ①