Gas Laws / (1) [Problems] — A Phys 5-21

- Gas Laws – 3 of them
- Ideal Gas Equation ... $PV = nRT$
- $PV = \frac{1}{3} Nm \langle v^2 \rangle$ or $P = \frac{1}{3}\rho \langle v^2 \rangle$
- Int. energy = $\frac{3}{2} nRT$
- 1 atm press = 100 kPa
- ρ_{air} = 1.2 kg/m³ at 1 atm, 25°C
- R (gas constant) = 8.3 J/K
- relative molecular mass of air = 28 (treat as nitrogen)
- rel. atomic mass of Helium = 4.
- rel. molecular mass of nitrogen = 28.
- Avogadro's number $N_A = 6 \times 10^{23}$.

1. This question is about the air in Room 12. – dimensions 20 × 10 × 3 m.

(a) Volume of air? (2)

(b) mass of air at 25°C, 1 atm.? (2)

(c) Number of.... (i) moles? (2) ...(ii) molecules? (2)

(d) At the start of a winter's morning the air temperature is 10°C. What is the air density now? (2)

(e) What is the total internal energy of the air ...
(i) at 10°C (ii) at 25°C. ?

(f) What is the r.m.s. velocity of the molecules at... (i) 10°C (ii) 25°C ?

2. The density of helium gas at 0°C and 1 atm. pressure is 0.18 kg/m³.

(a) What is the r.m.s. velocity of the atoms at... (i) 0°C (2) ... (ii) 25°C (2)

(b) What must the temperature become for the KE. and r.m.s. velocity to double from their values at 0°C? (4)

3 A balloon contains a mixture of helium (He) and hydrogen (H_2) gases at a uniform temperature. What are the ratios of ... (a) the mean K.E.
(b) the rms velocities
for the particles of each gas? (4)

4 A weather balloon is partially filled with 5 m³ of hydrogen at the ground at 10°C, 1 atm pressure. It is released and rises to a height of 10,000 m, where P = 0.1 atm, and T = –50°C. What is the volume of hydrogen now? (3)

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5. A double glazed window panel measures 1m × 1.5m with a gap of 16 mm, filled with nitrogen at 1 atm. pressure at 10°C

(a) How many moles? (2)

Between a min temp. of 0°C and a max. of 30°C, ...

(b) what is the difference in pressure? (3)

(c) what is the change in internal energy? (3).

6 This question is about some helium gas at 1 atm. and 20°C

(a) What is the volume of 1 mole of this gas? (2)

(b) How many moles of gas are in 1 m^3? (2)

(c) What is the internal energy in 1 m^3 of gas? (2)

(d) Show that the density of the helium is 0.166 kg/m^3 (2)

(e) How many atoms of helium are in 1 m^3? (2)

(f) what is the r.m.s. velocity of the helium atoms? (2)

(g) what is the mass of 1 helium atom? (2)

The 1 m^3 of He (at 1 atm. and 20°C) is compressed to 100 cm^3 and heated to 100°C

What is ... (h) the new pressure? (2)

(i) the new internal energy? (2)

(j) the new rms velocity? (2)

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Gas Law – ANSWERS /1

A Phy 1.4

Sheet 1. 1. (a) $V = 600\ m^3$ (b) mass $m = DV = 720$ kg 24258

$n =$

(c) (i) moles 1 mole has mass 28×10^{-3} kg so $720/28\times10^{-3} = 25{,}700$ moles

(or from $PV = nRT$ n moles at 25°C = 24,260 moles)

(ii) 1 mole has 6×10^{23} molecules so... 25,700 mole = 1.54×10^{28} molecules

(d) Cold 10°C morning... $n = PV/RT = 100\times10^3 \times 600 / 8.3 \times 283 = 25{,}540$ moles

so air density at 10°C = $25{,}540/24{,}260 \times 1.2 = 1.26\ kg/m^3$

(a bit trizksy.... air density 1.2 kg/m³ not fully consistent with $PV = nRT$)

from $PV = nRT$ air density at 1atm, 25°C should be 40.4 moles/m³ = 1.13 kg/m³.

(e) Int energy... (i) 10°C $= \frac{3}{2}\times 25{,}540 \times 8.3 \times 283$ (ii) 25°C

$= \frac{3}{2} nRT$ $= 90\times10^6$ J $= \frac{3}{2} \times 24{,}260 \times 8.3 \times 298$

$= 90 \times 10^6$ J

(comment: this is a constant volume problem: as the air is warmed its molecules move faster, (↑Int. E), but it expands, so there are fewer molecules in the room — these 2 cancel out so the Int. E. of the air in the room is constant, eg... If you double T (K) then Int. E is 2×, but you have ½ as many molecules)

2. (a) $P = \frac{1}{3} D \langle v^2 \rangle$ so $\langle v^2 \rangle = 3P/D = 1.667\times10^6 \Rightarrow V_{rms} = 1290$ m/s

This is for 0°C. We know that Int. Energy $(\frac{1}{2} m \langle v^2 \rangle) \propto T$

so for T increasing 273 K (0°C) to 298 K (25°C) an increase of 1.09×

the KE will go up by 1.09× and V_{rms} will go up by $\sqrt{1.09} = 1.044$

so V_{rms} at 25°C will be $1290 \times 1.044 \doteq 1350$ m/s

(b) for KE to double... T (in K) must double 273 K (0°C) to 546 K (273°C)

for V_{rms} to double then Int E. $(\frac{1}{2} m v^2)$ must quadruple

$(V_{rms} \propto \sqrt{KE})$ so new Temp must be $4 \times 273 = 1092$ K (819°C)

3. same Temp. → all particles have same mean K.E. so $\frac{KE\ (H_2)}{K.E.\ (He)} = 1.$

H_2 He

rel. mass 2 4

$KE = \frac{1}{2} m v^2$ – so if mass is halved, then v^2 must double and

V_{rms} must increase by $\sqrt{2}\times = 1.4\times$ so $\frac{V_{rms}\ (H_2)}{V_{rms}\ (He)} = 1.4$

4. constant mass (no. moles 'n') of hydrogen $= PV/RT = \frac{100\times10^3 \times 5}{8.3 \times 283} = 213$ moles

at altitude $V_{high} = nRT/P = 213 \times 8.3 \times 223 / 10\times10^3 = 39.4\ m^3$

(or $\frac{PV}{T}$ = constant. $P\downarrow \times \frac{1}{10}$th ; $T\downarrow \times \frac{223}{283} (= 0.79)$ ∴ $V \rightarrow V \times 10 \times 0.$ $= 39.4\ m^3$)

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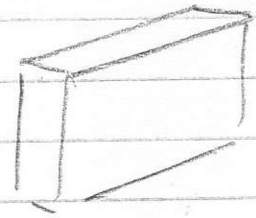
Gas Laws — Answers — 2 A Physics

5. (a) Vol = $1 \times 1.5 \times 16 \times 10^{-3}$

$= 0.024 \ m^3$



$PV = nRT \rightarrow n = \frac{PV}{RT} = \frac{100 \times 10^3 \times 0.024}{8.3 \times 283}$

$= \underline{1.02 \text{ moles}}$

(b) $P = \frac{nRT}{V}$ $\underline{0°C}$ $P_{0°C} = 96,500$ Pa (Mem. 1) $\underline{30°C}$ $P_{30°C} = 107,100$ Pa

∴ P diff, $\Delta P = \underline{10,600 \text{ Pa}}$

(c) Int.E = $\frac{3}{2} nRT$ $\underline{0°C}$ I.E. = 3473 J (mem.) $\underline{30°C}$ I.E. = 3854 J

So Difference in Int. Energy = $\underline{382 \text{ J}}$

6. (a) $PV = nRT$ V (1 mole) = $\frac{nRT}{P} = \underline{0.0243 \ m^3}$ /2

(b) n moles in $1 \ m^3$, = $1/0.024$ = $\underline{41.1}$ moles /2

(c) Int. E = $\frac{3}{2} nRT = \frac{3}{2} \times 41.1 \times 8.3 \times 293 = \underline{150 \text{ kJ}}$ /-

(d) density D (or ρ) = m/V = mass/1 = $n_{moles} \times$ molar mass = $\underline{0.164 \text{ kg/m}^3}$ /

(e) No. atoms $N = n \times N_A = 41.1 \times 6 \times 10^{23} = \underline{2.5 \times 10^{25}}$ /2

(f) $\underline{V_{rms}}$ $P = \frac{1}{3} \rho \langle v^2 \rangle \rightarrow \langle v^2 \rangle = 3P/\rho = 1.8 \times 10^6$

So $V_{rms} = \underline{1350}$ m/s /2

(g) mass He atom = rel. molar mass / N_A = $4 \times 10^{-3} / 6 \times 10^{23} = \underline{6.7 \times 10^{-27} \text{ kg}}$ /

Gas compressed + heated...

(h) Remember 1 m = 100 cm $PV = nRT$

$1 \ m^2 = 10^4 \ cm^2$ so $P_{new} = \frac{nRT}{V} = \frac{41.1 \times 8.3 \times 373}{100 \times 10^{-6}}$

$1 \ m^3 = 10^6 \ cm^3$

$= 1.27 \times 10^9$ Pa

(12.7 atm) /2

(or... if $T \rightarrow \frac{373}{273} \times T$, and $V \rightarrow \frac{10^2}{10^6}$,

then $P \rightarrow \frac{373}{273} \times \frac{10^4}{10^2} \times P = 1.27 \times 10^9$ Pa)

(i) I. Energy = $\frac{3}{2} nRT = \frac{3}{2} \times 41.1 \times 8.3 \times 373 = \underline{190 \text{ kJ}}$

(I.E. ∝ T so new I.E = $150 \times 373/293$ = 190 kJ) /2

(j) V_{rms} increased by factor of $\sqrt{190/150} = 1.13$

So new $V_{rms} = 1350 \times 1.13 = \underline{1520 \text{ m/s}}$ /2

(or new density = $0.164/100 \times 10^{-6}$ = 1640 kg/m³, use $P = \frac{1}{3} \rho \langle v^2 \rangle$ to show V_{rms} = 1520 m/s)

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