TLW

Circular Motion – 1/5

Lots of examples!

in all examples $g_{Earth} = 9.8\ N/kg$

♫ The Sun has got his hat on... ♫ #

A2 Physics

1. Earth + Sun

(a) Draw a labelled diagram showing all the forces acting on the Earth (2)

Calculate...

(b) The frequency of the Earth's orbit in Hz (2) $r_{Earth} = 1.5 \times 10^{8}$ km

(c) Earth's linear speed (2) $M_{Earth} = 6.0 \times 10^{24}$ kg

(d) Angular velocity round the Sun (2) $T_{Earth} = 365$ days

(e) Centripetal acceleration (2) Unleaded = $79.5\ p\ l^{-1}$

(f) Calculate the centripetal force on the Earth, and state what produces this force (3)

(g) "Gravitational attraction obeys an Inverse Square Law." What does this statement mean? (2)

(h) If the Earth's orbit was twice the radius, what would the gravitational attraction from the Sun be? And what would be the Time period of the new orbit? (4)

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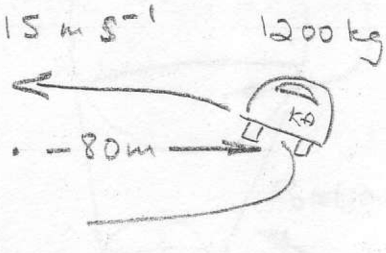
2. Going round the Bend

A car is driving at a steady speed round a circular track.



(a) Draw a diagram showing the forces on the car (2)

(b) What is the car's angular velocity? (2)

(c) What is the centripetal force on the car, and what provides it? (explain this last bit carefully!) (4)

(d) If the maximum friction force is 2.8 kN, what is the car's maximum speed without skidding? (2)

(e) ... and what is the Time period for 1 complete circle? (2)

(f) The car is now driven on an experimental road surface where the maximum friction force is 35% of the vehicle's weight. What is the maximum speed now? (3)

(g) For the same road surface, how does the maximum speed depend on the radius of the curve? (2)

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Q... (1) /19 ... (2) /17

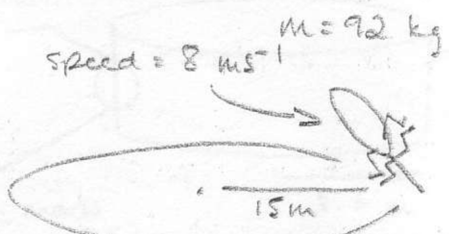
TLW 07.04.

Circular Motion – 2/5

Remember $g = 9.8\ \mathrm{N\,kg^{-1}}$!!

3 On yer Bike!

A cyclist is travelling in a circular path ⇨



(a) Draw a diagram to show the forces on the rider + bike – treat them as 1 unit. (2)

(b) Calculate the upwards reaction force on bike + rider. (2)

(c) Calculate centripetal force on bike + rider (3)

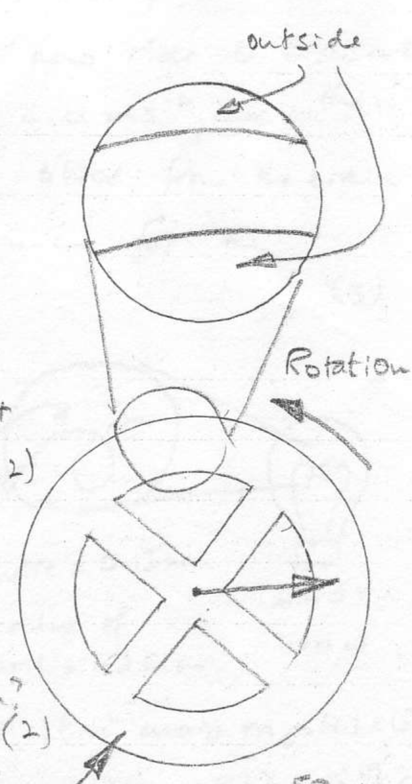
(d) From (b) and (c) calculate the angle of bike + rider to the vertical (3)

(e) If the bike + rider are at an angle of 35° to the vertical what is their speed on the circular path? (3)

(f) If the maximum angle to the vertical is 40° for the tyres just not to skid, what is the maximum friction force as a fraction of the weight of bike + rider? (2)

4. Lost in Space.

The diagram shows a space station shaped liked a doughnut (but without the jam). It is rotating to produce an artificial gravity field of 10 N/kg at the rim.



(a) Draw a diagram to show where an astronaut stands, and the direction of artificial gravity (2)

(b) Explain how the articial gravity is created (2)

(c) What angular velocity is needed to produce a gravity field strength of 10 N/kg at the rim? (2)

(d) What is the rim velocity? (2)

(e) What is 'g' half way along a radial arm? (1)

(f) Is there anywhere on the station where g = 0? (1)

(g) Why doesn't an astronaut at the rim feel the station rotating? (1)

(h) An astronaut steps outside the space station and forgets to hold on. Draw a diagram to show the path he/she takes (1)

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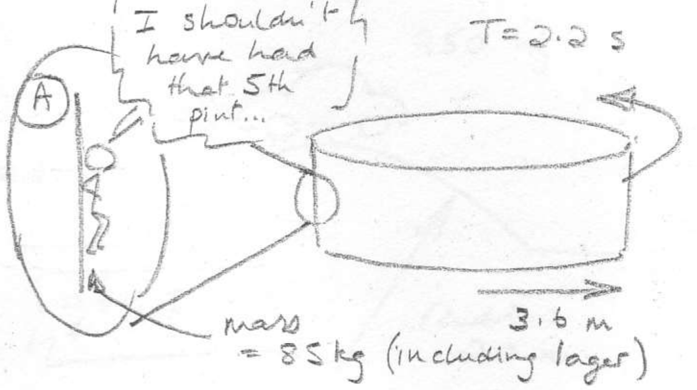
RW
07.04

Circular Motion - 3/5



5 Wall of Death

The riders spin round inside a cylindrical cage

(a) One rider explains..."The centrifugal force throws you against the wall of the cage". Explain how this statement is wrong. (2)

(b) Copy diagram A and show the forces on the rider - you can add words of your own (2 + 1 for words)

(c) What force stops the rider falling? (1)

Calculate... (d) the rider's linear velocity (2)

(e) Centripetal force (3)

(f) Show that the ratio of centripetal force to the rider's weight is.... $\omega^2 r/g$. (1)

(g) The owner wants to build a bigger ride, with a 5 m radius, attract more customers, and retire early. He knows that the ratio in (f) must be at least 2.0 for safety.

What is the longest time period for the new ride to achieve this? (3)

(h) If the centripetal acceleration gets above 4.0 ms^{-2}, then riders can "black out" due to loss of blood from the brain.

Calculate the time period and frequency for the fastest safe spin of the bigger ride. (3)

6. Light up my life!

The filament in a light bulb is coiled as shown ⟹

$d_{wire} = 0.13$ mm

240 V

radius of coil = 1.2 mm

100 W

electron mass $m_e = 9.1 \times 10^{-31}$ kg

electron charge $e = 1.6 \times 10^{-19}$ C

resistivity of tungsten = 5×10^{-8} Ωm

tungsten has 1×10^{28} free electrons/m^3.

10.6/18

(a) Calculate the electrons' drift velocity in the filament (4)

'25/18

(b) Calculate the centripetal acceleration of an electron in the filament, (2)

(c) What force is needed to keep the electron in it's circular path? (2)

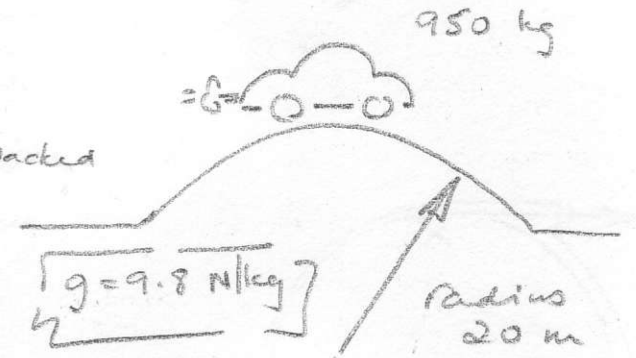
T2W

07.04

Circular Motion – 4/5

7 Bog Racer

A car drives over a hump-backed bridge, with a radius of 20m.



g = 9.8 N/kg

(a) Draw a diagram of the forces on the car at the centre of the bridge (2)

(b) What force provides the centripetal force to keep the car moving in a circle? (1)

(c) Calculate the maximum velocity the car can have just to maintain contact with the road at the centre of the bridge (3)

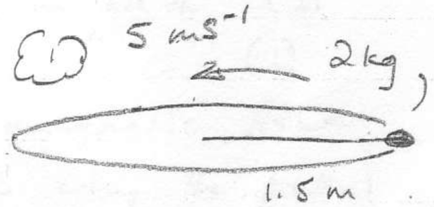
State and explain the effect on the maximum velocity if…

(d) … the mass of car + occupants is increased by 50% (2)

(e) … the radius of the bridge is halved (2)

8. David vs. Goliath

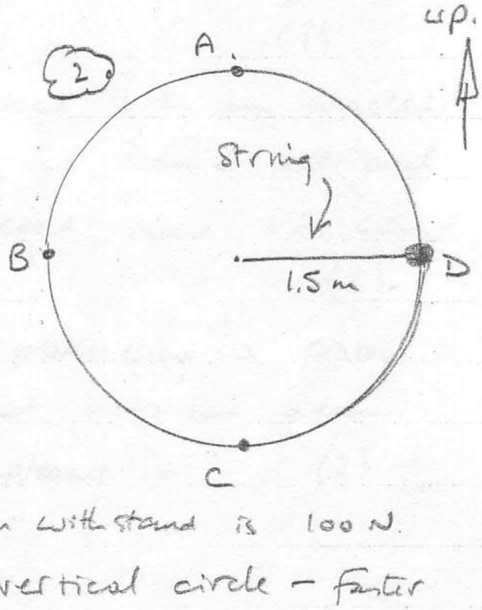
(a) A 2kg stone is swung in a horizontal circle – diagram (1)



Calculate the tension in the string (2)

(b) Explain why a constant tension is needed (3)

The stone is now swung in vertical circle



(c) Draw diagrams of the forces on the stone at points A, B and C. (3)

(d) Calculate the string tensions at points A, B and C ($3 \times 2 = 6$)

10/20

(e) The maximum tension the string can withstand is 100 N. The stone is swung – still in a vertical circle – faster until the string breaks.

5/10

Draw a diagram to show where the stone is in the circle when the string breaks, and the path of the stone (2)

(f) Calculate the stone's velocity and time period when the string breaks (4)

TJN
07.04

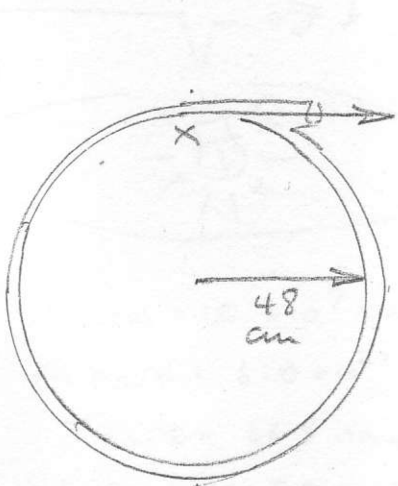
Circular Motion - 5/5

 phew!

9. Smashing atoms!

A cyclotron produces high speed particles by accelerating them in a circular path of steadily increasing radius.

The final circular path has a radius of 48 cm, and the particles emerge with a velocity 75% of the speed of light.



(a) Calculate .... angular velocity, frequency and time period in the final circular path (5)

m proton = m neutron = 1.6×10^{-27} kg

$e = 1.6 \times 10^{-19}$ C

V light = 3×10^{8} m s^{-1}

(b) The particles being accelerated are protons. Calculate the centripetal force on each (2)

(c) What is the K.E. of each proton? (1)

(d) The centripetal force is provided by a magnetic field which is "switched off" at X. Explain why the protons take the path shown. (1)

(e) If you wanted to produce high speed Lithium nuclei ($^{7}_{3}Li$) and you could only produce the same centripetal force as in (b), what maximum speed and K.E. could you achieve? (3).

(f) The cyclotron is described as producing "a proton beam current of 6×10^{-9} A". What does this mean? How many protons/second does this represent? (3)

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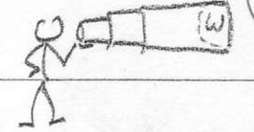
TJW
07.04

Circular Motion ANSWERS



T2N

1/

1\. (a)

(2)

(b) $f = 1/T$ ← in sec

$= 1 / 365 \times 24 \times 60 \times 60$ ①
d h m s

$= \underline{3.2 \times 10^{-8}}$ ① Hz. (2 s.f.) /2

(c) $v = d/t = 2\pi r/t$ ① ← metres !!

$= 2\pi \times 1.5 \times 10^{11} / 31.5 \times 10^6$ (s.)

$= 30{,}000$ m/s $= 3.0 \times 10^4$ m/s ① (2 s.f.) /2

(d) $v = \omega r \Rightarrow \omega = v/r$ ① $= 3 \times 10^4 / 1.5 \times 10^{11} = \underline{0.2 \times 10^{-6}}$ ① rad s^{-1} /2

(e) $a_c = v^2/r$ ① $= (3 \times 10^4)^2 / 1.5 \times 10^{11} = \underline{0.0060}$ ① m/s^2 $= 6.0 \times 10^{-3}$ ms^{-2} (2 s.f.) /2

(f) $F_c = ma_c$ ① $= 6.0 \times 10^{24} \times 0.006 = \underline{3.6 \times 10^{22}}$ N ①

gravitational attraction from the Sun. ① /3

(g) Inverse □ Law... If you double the distance from the Sun, the gravitational attraction reduces to 1/4 ($1/2^2$)

Force of gravity $F_g \propto 1/d^2$ /2

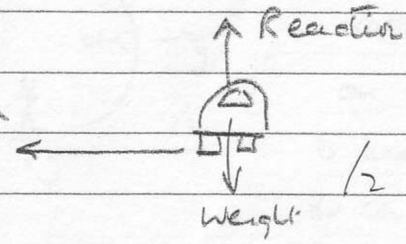
(h) 2× earth's radius.... so. $F_g = 3.6 \times 10^{22}/4 = \underline{9 \times 10^{21}}$ N ①

HARD!! So $F_c = m\omega^2 r$ ① $\Rightarrow \omega^2 = F_c/mr = 9 \times 10^{21} / 6 \times 10^{24} \times 3 \times 10^8$

$= 5 \times 10^{-16}$

So $\omega = \underline{2.2 \times 10^{-6}}$ ① rad/s $T = 2\pi/\omega = \underline{2.8 \times 10^6}$ ① Sec

2\. (a)



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(b) $v = \omega r \Rightarrow \omega = v/r$ ① $= 15/80$

$= \underline{0.19}$ ① rad/s. /2

(c) $F_c = \frac{mv^2}{r}$ ① $= 1200 \times 15^2/80 = \underline{3380}$ ① N (3 s.f.). Centripetal force is provided by the friction force of the road on the tyres. ② /4

(d) $F_c = mv^2/r \Rightarrow v^2_{max} = F_{c\,max} \times r/m$ ① $= 2800 \times 80/1200 = 187$

So $v_{max} = \underline{13.7}$ ① m/s (3 s.f.) /2

(e) $2\pi r = vT \Rightarrow T = 2\pi r/v$ ① $= 2\pi\, 80 / 13.7 = \underline{36.8}$ s ① /2

(f) $f_{max} = 0.35 \times mg$ ① $= 0.35 \times 1200 \times 9.8 = 4120$ N. ①

$F_c = mv^2/r \Rightarrow v^2_{max} = F_c \times r/m = 4120 \times 80/1200 = 274$

So v_{max} now $= \sqrt{274} = \underline{16.6}$ ① ms^{-1} (check against answer (d)) /3

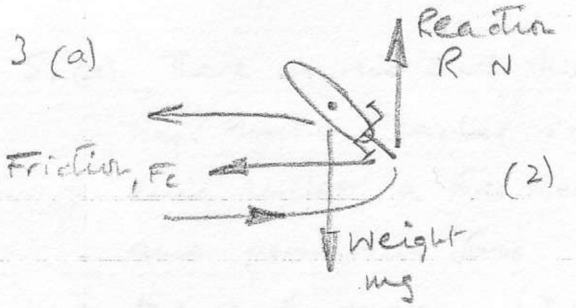
(g) getting harder... same road ∴ constant $F_c = mv^2/r$

So $v^2 = F_c \times r/m$ so $v^2 = k \times r$ k = constant $= F_c/m$

So $v = \sqrt{k} \times \sqrt{r}$ or $v \propto \sqrt{r}$... v_{max} is proportional to $r^{1/2}$. /2

Circular Motion — Answers — 2/5

Q. 3 (a)

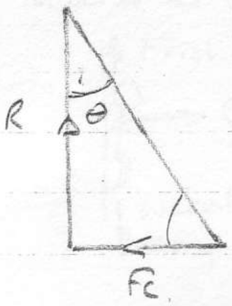


(2)

(b) $R = mg = 92 \times 9.8 = \underline{902}$ N (3 s.f.) /2

(c) $F_c = mv^2/r = 92 \times 8^2/15$
$= \underline{393}$ N /2

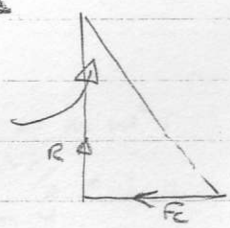
(d)



Draw Δ of forces – forces follow each other round the Δ

so $\tan\theta = F_c/R = \frac{393}{902} = 0.436 \Rightarrow \theta = 23.5°$ (to vertical) 3 s.f. /3

(e) angle of 35°

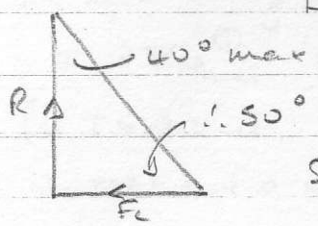


$\tan 35 = F_c/R$ so... $F_c = R \tan 35 = 902 \times \tan 35$
$= \underline{632}$ N

$F_c = \frac{mv^2}{r} \Rightarrow v^2 = \frac{F_c \times r}{m} = \frac{632 \times 15}{92} = 103$

so $v_{max} = \sqrt{103} = \underline{10.1}$ ms^{-1} /3

(f)



$\tan 40 = \frac{F_c}{R} \Rightarrow F_c = R \tan 40 = 0.84 \times R$

So F_c max = weight of (bike + rider) × 0.84 /2

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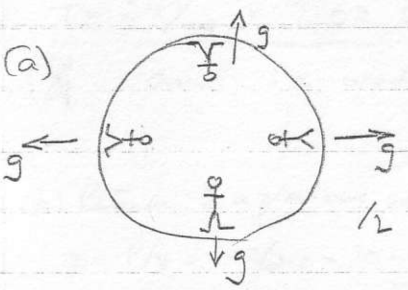
4. (a)



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(b) The ~~doughnut~~ space station is rotating.... the "floor" presses "upwards" on the feet... providing the centripetal force to keep the astronaut accelerating towards the centre... this "feels" like gravity. /2

(c) $g = 10$ Nkg^{-1} $= a_c = \omega^2 r$ so $\omega = \sqrt{10/50} = \underline{0.447}$ rad s^{-1} /2

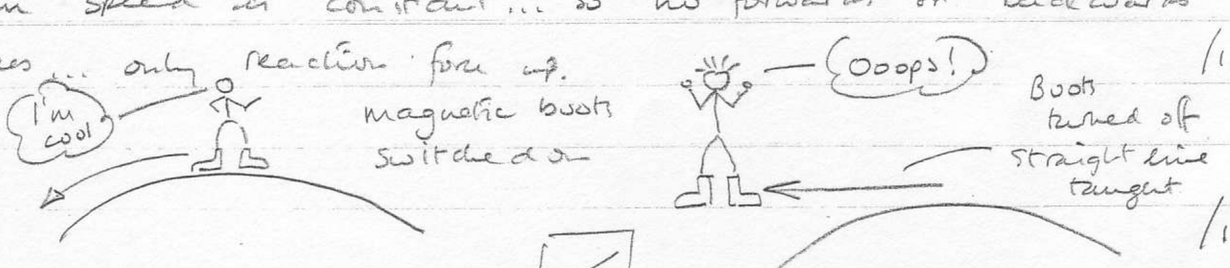
(d) Hmm... remember $v = \omega r = 0.447 \times 50 = \underline{22.3}$ ms^{-1} 3 s.f. /2

(e) Halfway along $g = a_c = \omega^2 r = 0.447^2 \times 25 = \underline{5.011}$ ms^{-2} /1

(f) yes... at the centre... $a_c = \omega^2 r$ if $r = 0$ $a_c = 0$
You just rotate... no centripetal acceleration... you're at the centre! /1

(g) Run speed is constant... so no forwards or backwards forces... only reaction force up. /1

(h)



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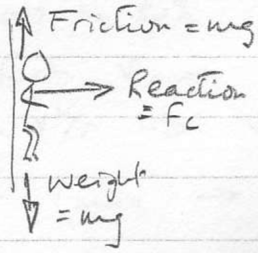
Circular Motion *Answers* 3/5

5. (a) "There is no such thing as centrifugal force" – this must be correct, my Physics Teacher says so*... moving objects travel in a straight line unless a force acts on them (1) ... the wall travels in a circle and produces a force on the rider, accelerating (1) him/her/it towards the centre. (*sadly... no marks for giving this as a reason!) /2

(b)



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(c) Friction of the wall on the rider /1

(d) $\omega = 2\pi/T = 2.86$ (1) rad s^{-1}

$v = \omega r = 2.86 \times 3.6 = \underline{10.3}$ (1) m s^{-1} 3 s.f. /2

(e) $F_c = m a_c$ (1) $= m\omega^2 r$ (1) $= 2500$ (1) $\text{N} = \underline{2.5 \times 10^3\ \text{N}}$ 2 s.f. /3

(f) $F_c = m a_c = m\omega^2 r$ .... weight = mg ... so $\frac{F_c}{\text{weight}} = \frac{m\omega^2 r}{mg} = \frac{\omega^2 r}{g}$. (1)

(g) we need $\omega^2 r/g = 2.0$ so $\omega^2 = 2.0 \times g/r = 3.92$ so $\omega_{min} = \underline{1.98}$ (1) rad s^{-1}

$T = 2\pi/\omega_{min}$ (1) so $T_{max} = 2\pi/1.98 = \underline{3.2}$ (1) sec (2 s.f.) /3

(If T > 3.2 sec ... ride is too 'slow' ... F_c not enough ... rider slides down)

(h) $a_c\ max = 4.0\ \text{ms}^{-1} = \omega^2 r$ ... so $\omega^2_{max} = 4/5 = 0.8$ ... so $\omega_{max} = \underline{0.89}\ \text{rad s}^{-1}$

$T = 2\pi/\omega = \underline{7.02\ \text{s}}$ ... and $\omega = 2\pi f$ so $f = \omega/2\pi = \underline{0.14\ \text{Hz}}$ /3

(* ... and I've made a small mistake ... it should have been $a_c\ max = 40.0\ \text{ms}^{-2}$)

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6. (a) Return to a previous existence $I = nAve \Rightarrow v = I/nAe$

$I = P/V = 100/240 = \underline{0.417\ A}$ (1) $A = \pi r^2 = \pi \times \left(\frac{0.13 \times 10^{-3}}{2}\right)^2 = \underline{1.33 \times 10^{-8}}$ (1) m^2

so $v\ drift = 0.417 / 1 \times 10^{28} \times 1.33 \times 10^{-8} \times 1.6 \times 10^{-19}$ (1) $= 0.0196\ \text{ms}^{-1}$ (1) $= \underline{1.96 \times 10^{-2}\ \text{ms}^{-1}}$ /4

(b) $a_c = v^2/r$ (1) $= (1.96 \times 10^{-2})^2 / 1.2 \times 10^{-3}$

$= \underline{0.321}$ (1) m s^{-2} /2

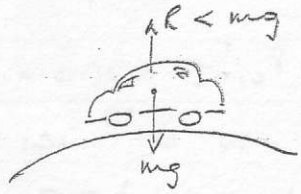
(c) $F_c = m a_c$ (1) $= 9.1 \times 10^{-31} \times 0.321$

$= \underline{2.9 \times 10^{-31}}$ (1) N (2 s.f.) /2

8

Circular Motion Answers

7. (a)



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(b) Gravity (weight) keeps the car moving in a circle /1

(c) when just in contact with the road ... then R ↑ = 0.

So $F_c = \cancel{m}g = \cancel{m}v^2/r$ ① $v_{max} = (gr)^{1/2}$ ① $= (9.8 \times 20)^{1/2} = \underline{14}$ ① ms^{-1} /3

(d) ↑ mass by 50% ... $mg = mv^2/r$ so changing mass does not affect v_{max} /2

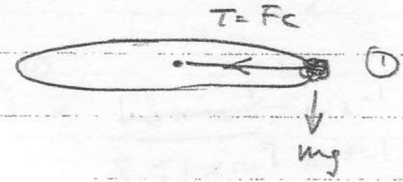
(e) $g = v^2/r \Rightarrow v_{max} = \sqrt{g} \times \sqrt{r}$ ... so if $r \rightarrow \frac{1}{2}r$ then $\sqrt{r} \rightarrow \frac{\sqrt{r}}{\sqrt{2}}$

So $v_{max} \rightarrow v_{max}/\sqrt{2}$ /2

/10

8. (a) Horizontal circle ... (3 s.f.)

$T = F_c = mv^2/r = 2 \times 5^2/1.5 = \underline{33.3}$ N (2)



(b) Constant tension needed .... to keep the stone constantly <u>accelerating</u> towards the <u>centre</u> of the circle. <u>Weight acts at 90°</u> to the string, so doesn't affect the tension. /3

(c) diagram ⟶ /3

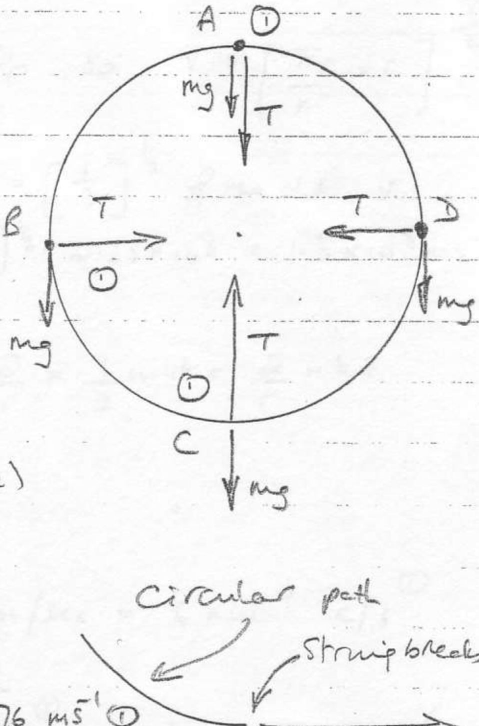
(d). at A... $F_c = T + mg$

So $T = F_c - mg = 33.3 - (2 \times 9.8) = \underline{13.7 \text{ N}}$ ②

at B $T = F_c = \underline{33.3}$ N ②

at C $T = F_c + mg = 33.3 + (2 \times 9.8) = \underline{52.9 \text{ N}}$ ② /6

(F_c is the centripetal force to maintain the circular motion – so $F_c = \frac{mv^2}{r} = 33.3$N constant).



(e) See diagram ⟶

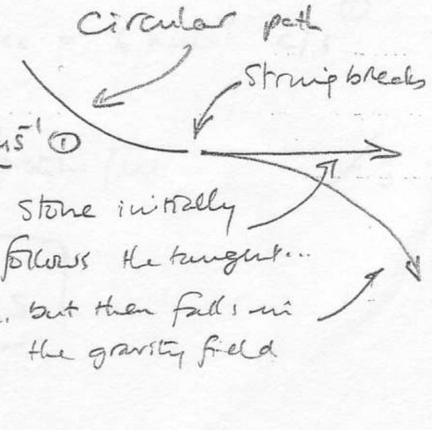
String breaks when stone at the bottom. (2)

(f) $T = F_c + mg = 100$ N max

So $F_c = 100 - mg = 100 - (2 \times 9.8) = 80.4$ N ①

So $mv^2/r = 80.4$ ① So $v = \left(\frac{80.4 \times 1.5}{2}\right)^{1/2} = \underline{7.76 \text{ ms}^{-1}}$ ①

$2\pi r = v \times T$ so $T = 2\pi r/v = 2\pi \times 1.5/7.76 = \underline{1.2}$ s ①



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Circular Motion   Answers   5/5

9. (a) $v = 0.75 \times 3\times10^{8} = \underline{2.25\times10^{8}}$ (1) $\mathrm{m\,s^{-1}}$

$v = \omega r$ (1) so $\omega = v/r = 2.25\times10^{8}/48\times10^{-2} = \underline{4.69\times10^{8}}$ (1) $\mathrm{rad\,s^{-1}}$

$T = 2\pi/\omega = \underline{1.34\times10^{-8}}$ s (1)

$f = 1/T = \underline{7.46\times10^{7}}$ Hz (1)   /5

(b) $F_c = mv^2/r$ (1) $= 1.6\times10^{-27}\times(2.25\times10^{8})^2/0.48 = \underline{1.69\times10^{-10}}$ N (1)   /2

(c) $KE = \frac{1}{2}mv^2 = \frac{1}{2}\times 1.6\times10^{-27}\times(2.25\times10^{8})^2 = \underline{4.05\times10^{-11}}$ J   /1

(d) when the magnetic centripetal force stops acting, the particles continue in a straight line.   /1

(e) $F_c = 1.69\times10^{-10} = mv^2/r$

!! $m_{Li\ ion} = 7\ \cancel{3}\times 1.6\times10^{-27} = \cancel{4.8\times10^{-27}}$ kg 1.12×10^{-26} kg

nucleon number!

So $v_{max} = (F_c \times r/m)^{\frac{1}{2}}$

$= \left(1.69\times10^{-10}\times 0.48 / \cancel{4.8\times10^{-27}}\ 1.12\times10^{-26}\right)^{\frac{1}{2}}$ (1) $= \cancel{1.3\times10^{8}}$ (1) $\mathrm{m\,s^{-1}}$

$\underline{8.51\times10^{7}\ \mathrm{m\,s^{-1}}}$ /3

☆ Check this is < v of protons!

So KE of Li ions $= \frac{1}{2}mv^2 = \frac{1}{2}\times 4.8\times10^{-27}\times(1.3\times10^{8})^2 = \underline{4.06\times10^{-11}}$ J

[Think ... (1) calculating v_{Li} ... $F_c = mv^2/r$ so $v = \left[\frac{F_c}{m}\times r\right]^{\frac{1}{2}}$

So if m is trebled (3×) then new $v = \left[\frac{1}{3}\right]^{\frac{1}{2}}$ of the old v

So $v_{Li} = (1/3)^{\frac{1}{2}} \times v_{proton} = (0.33)^{\frac{1}{2}}\times 2.25\times10^{8} = 1.3\times10^{8}\ \mathrm{m\,s^{-1}}$

and (2) $F_c = mv^2/r = \frac{1}{r}\times mv^2 = \frac{2}{r}\times\frac{1}{2}mv^2 = \frac{2}{r}\times KE.$

So k.e. must stay the same!]

(f) proton beam current I = no. of Coulombs/sec $= 6\times10^{-9}$ C/s (1)

each proton has a charge of 1.6×10^{-19} C

So no. of protons/sec $= \dfrac{6\times10^{-9}}{1.6\times10^{-19}}$ (1) $= 3.8\times10^{10}$ (1) protons/sec.   /3

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